

(59)

$$\lim_{x \rightarrow 0} x \sin x$$

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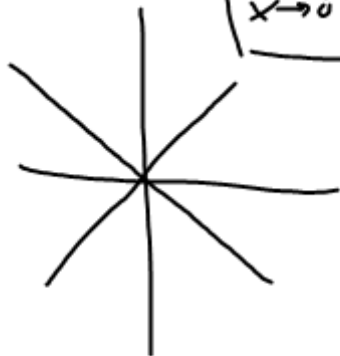
$$-1 \leq \sin x \leq 1$$

$$-x \leq x \sin x \leq x$$

$$\lim_{x \rightarrow 0} -x \leq \lim_{x \rightarrow 0} x \sin x \leq \lim_{x \rightarrow 0} x$$

$$0 \leq \lim_{x \rightarrow 0} x \sin x \leq 0$$

$$\boxed{\lim_{x \rightarrow 0} x \sin x = 0}$$



(24)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} \cdot \frac{2}{2} = \lim_{x \rightarrow 0} 2 \frac{\sin 2x}{2x}$$

$$= 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$$

$$= 2 \cdot 1$$

$$\boxed{= 2}$$

$$(22) \quad \lim_{x \rightarrow 0} \frac{\frac{2}{2+x} - \frac{1}{2} \cdot \frac{2+x}{2+x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{2 - (2+x)}{2(2+x)}}{x} \Rightarrow \frac{\frac{\cancel{x}}{4+2x}}{x} \cdot \frac{1}{\cancel{x}} = \frac{1}{4+2x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{4+2x}$$

$$\boxed{= \frac{1}{4}}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$$

60

$$\lim_{x \rightarrow 0} x^2 \sin x$$

$$-1 \leq \sin x \leq 1$$

$$-x^2 \leq x^2 \sin x \leq x^2$$

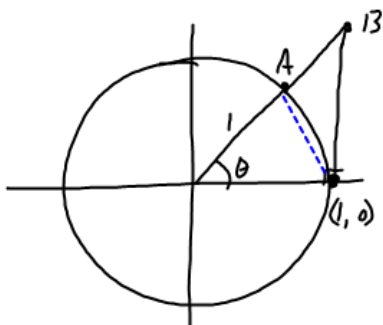
$$\lim_{x \rightarrow 0} -x^2 \leq \lim_{x \rightarrow 0} x^2 \sin x \leq \lim_{x \rightarrow 0} x^2$$

$$0 \leq \lim_{x \rightarrow 0} x^2 \sin x \leq 0$$

$$\lim_{x \rightarrow 0} x^2 \sin x = 0$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

— Visualize a sector squeezed between 2 triangles



$$A(\cos \theta, \sin \theta)$$

$$B(1, \tan \theta)$$

The sector is less than one Δ bigger than the other

$$\frac{\sin \theta}{2} \leq \frac{\theta}{2} \leq \frac{\tan \theta}{2}$$

mult by $\frac{2}{\sin \theta}$

$$1 \leq \frac{\theta}{\sin \theta} \leq \frac{1}{\cos \theta}$$

Take reciprocal, switch inequalities

$$1 \geq \frac{\sin \theta}{\theta} \geq \cos \theta$$

Use Sandwich Theorem and take limit

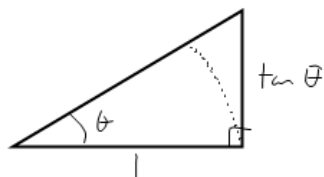
$$\lim_{x \rightarrow 0} \left(1 \geq \frac{\sin \theta}{\theta} \geq \cos \theta \right)$$

$$\lim_{x \rightarrow 0} 1 \geq \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} \geq \lim_{x \rightarrow 0} \cos \theta$$

↓

$$1 \geq \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} \geq 1$$

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$



Area

$$\frac{\tan \theta}{2}$$

$$\frac{\theta}{2} \left[\frac{\theta}{2\pi} (\pi r^2) = \frac{r^2 \theta}{2} \right]$$

$$\frac{\sin \theta}{2}$$